A Bayesian Method for Managing Uncertainties Relating to Distributed Multistatic Sensor Search

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Abstract – Predicting the search effectiveness of a distributed multistatic sensor field is highly conditioned on information which is unknown and, for all practical intents, unknowable when engaged in a two-sided tactical situation. Yet, it is imperative to have a method for assessing the military value of such systems to inform decisions relating to procurement, optimal employment, and maximal military exploitation. The combination of Monte Carlo simulation methods and Bayesian fusion techniques allow for a robust approach for modeling the effects of uncertainty on the distribution of likely outcomes. Exemplar analysis for an Area Clearance and an Area Denial scenario demonstrate how a combined Monte Carlo simulation and Bayesian fusion system might be employed to account for uncertainty and the types of information products they can provide a decision-maker.

Keywords: Bayesian; Multistatic Operations Analysis; Decision Support, Situational Assessment, Uncertainty

1. Introduction

There are many uncertainties that impact the reliability of any deterministic sonar system performance prediction. Even in the simple and long-studied domain of passive sonar systems, much can be said about the difficulty to match observed performance with modeled performance. Monostatic active sonar adds new complexity to solving the sonar equation with the addition of two-way propagation loss and a new acoustic driver, reverberation. Disregarding the potential difficulties, many look to multistatic sonar as a means to increase search effectiveness. There are attractive first principles motivating this idea. Importantly, there are increased opportunities for a high value target strength or a high Doppler geometry that an enemy submarine is not able to control, because it does not know the location of all the receivers processing the echo. The extra receivers processing each ping and high target strength and high Doppler opportunities give rise to the expectation of an increased number of detections that could feed a track-before-detect process. There will also be an increased probability of high signal-to-noise ratio (SNR) detections associated with specular and near-specular detections that aid in distinguishing a submarine from clutter. Lastly, the construction of a multistatic field having separate sources and receivers allows for a cost-based optimization for field design that reflects the disparity in cost between sources and receivers.

2. Cataloguing the Uncertainties

While multistatic sensors offer the promise of improved detection opportunity, they do not alleviate the uncertainties intrinsic to assessing field performance. There are many uncertainties that may impact the bistatic sonar equation, formula 1 [1]. This paper catalogues some of the uncertainties that remain even after the range dependent environment is well surveyed, a reliable range dependent acoustic propagation model is selected, the target is well researched and the sonar system parameters ascertained and verified.

$$SE = SL - TL_{S\text{-}T} - TL_{T\text{-}R} - (AN - DI) \oplus RVB + TS - DT \quad \textbf{(1)}$$

Where:
SE = signal excess
SL = source level
$TL_{S\text{-}T}$ = transmission loss source to target
$TL_{T\text{-}R}$ = transmission loss target to receiver
AN = ambient noise (omnidirectional noise)
DI = directivity index
RVB = reverberation in the beam
TS = target strength
DT = detection threshold
Note – This equation can be solved in the energy domain or in the power domain.

2.1 Target Motion Behavior

Prior to detection, enemy submarine motion in three dimensional space is not knowable. The uncertainties include not knowing the target motion objectives, its depth operating profile, or its speed operating profile. But not knowing these important parameters does not mean that field performance assessment is precluded. What is necessary is to have estimates of credible target motion parameters that as a minimum include the possibility of aggressive behavior (e.g., driving towards friendly forces). If all credible behaviors can be identified, then they can be modeled and evaluated as concurrent, competing hypotheses. Metrics against each of these hypotheses can be evaluated to assess the most robust tactic in the presence of tactical uncertainty.

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2.2 Target Aspect

Target aspect with respect to a source and nearby receivers in the field is a discrete phenomenon; that is to say that a real target will only be in one location and have one heading, depth and speed at any given time. The problem is that its exact location, heading, depth and speed at an exact moment in time are somewhat arbitrary. Unfortunately, the field performance against that particular analytical instantiation is extremely sensitive to those parameters. One way to accommodate for this is to use a sufficiently large numbers of Monte Carlo simulations to represent the distribution of likely geometries that may be encountered. Target orientation will determine the value of target strength, TS in formula 1. Figure 1 shows how the target strength of a submarine hull may vary as a function of bistatic angle and bistatic aspect angle. The Bistatic Angle is the source-target-receiver angle. The Bistatic Aspect Angle is the angle on the bow of the bisector of the Bistatic Angle. Target orientation and speed will determine the value of reverberation, RVB in formula 1, once various Doppler filters are applied to the returned signal. Figure 2 shows how the sonar system may be able to reject reverberation as a function of bistatic Doppler. Target depth, which may be variable, will determine the selection of an appropriate transmission loss calculation, the TL terms in formula 1.
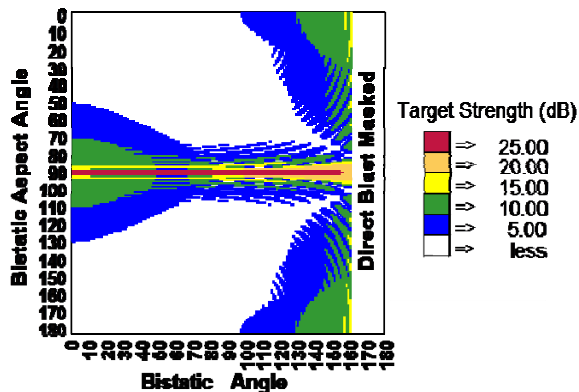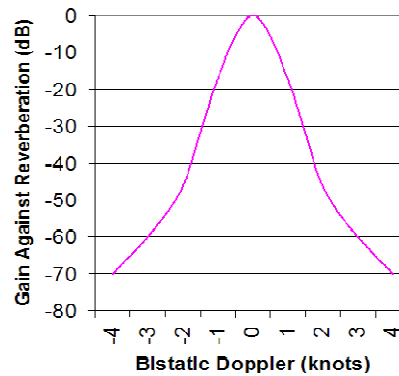

Figure 2. Notional Representation of Reverberation Rejection as a Function of Bistatic Doppler

2.3 Detection Uncertainty

Experience to date has taught us that it is futile to attempt to monitor our environment and other performance related parameters in the hopes of being able to predict (model) accurately which pings will provide detection and which ones will not. For example, given 20 acoustic "looks" each having a probability of 0.1 of generating a detection, there is no reliable means of anticipating which of the ten percent opportunities will pay-off. Indeed, it is possible that none of them will.

2.4 Acoustic Fluctuation

Even when the signal excess equation, Formula 1, is supplied with the best available estimates for each term, there will be fluctuations in detection performance driven by fluctuations in the environment [2]. These environmental fluctuations often cannot be measured and their acoustic effect cannot be modeled. As an example, we may be able to anticipate the presence of internal waves, but measuring their structure throughout a large volume is prohibitively difficult. Even if we could measure their structure, we are not yet able to anticipate the induced acoustic effect of the internal waves on a particular source-receiver pair as they look upon a target. Yet, these fluctuations can have a large effect on sensor performance, and so must be accounted for by some means. [3]

Our current models, databases, environmental sensing, and oceanographic tools enable us to reasonably predict median environmental conditions and thus median sensor performance. Working with median values, we can arrive at a deterministic calculation of system performance. Because of the acoustic fluctuations, the actual performance of the system may be greatly improved over or greatly diminished from median, deterministic expectations. The acoustic fluctuation is often represented as a stochastic process, having a time scale. Some fluctuations happen on the time scale of ping-to-ping, while other fluctuations occur

Figure 1. Notional Representation of Target Strength as a Function of Bistatic Angle and Bistatic Aspect Angle.

over the course of hours. There are multiple frameworks for accounting for these fluctuations [4] [5] [6], including a lambda-sigma model and a Gauss-Markov process. Each has its merits and application. A detailed discussion of these two methods and their relative merit are outside the scope of this paper. What is important is that fluctuations are accounted for in the performance prediction model.

$$SE(t) = \overline{\overline{SE}}(t) + \xi(t) \qquad (2)$$

Where:

$SE(t)$ - modeled signal excess that combines the deterministic and stochastic components

$\overline{\overline{SE}}(t)$ - deterministic contribution to signal excess arrived at by solving Formula 1

$\xi(t)$ - stochastic contribution to signal excess

By example, consider a barrier search having a median performance of multiple detections on a transiting submarine. One applies considerations for acoustic uncertainty (i.e., converting signal excess into probabilities of detection) and assuming a Poisson distribution of detection events, determines that there is virtually no chance of the submarine transiting undetected. When fluctuations are ignored, this barrier search tactic would be deemed effective. Next we consider the same barrier search scenario, but introduce fluctuation modeling. When an unfortunate "draw" is made, either for a lambda-sigma or Gauss-Markov process, barrier search performance is degraded and the submarine may be able to transit the field undetected. So far, the above could be analyzed with analytic models if one were provided exact source-target-receiver geometries for each ping event and other pertinent data like target depth, speed, and heading. But, because the enemy submarine is not constrained to a single track or a single operating profile, analytic solutions are not an appropriate approach. The exemplar analyses contained in this paper employ a fluctuation model that have two long term Gauss-Markov processes and one short term Log-Rayleigh process.

3. Attributes of Monte Carlo and Bayesian Method

This section identifies the two central engines of the Multi-Sensor Interaction Calculator (MUSICAL™), which is used to generate the exemplar analyses.

3.1 Monte Carlo Simulation

So far in the paper, we have seen that there are many conditions that drive sensor performance expectations even after the environment is well surveyed and reliable acoustic models are used to predict performance for well known sensors against well understood targets. In a two-sided tactical scenario, the enemy submarine depth, speed, and motion strategy cannot be known. The actual location and heading of the target at any given time prior to detection is unknowable. Since these unknowable data drive sensor performance, what is needed is a means to instantiate a statistically significant representation of all credible behaviors. Monte Carlo simulation methods are well suited for this as long as the number of states is kept to a manageable level. When the number of states is allowed to grow, there is a combinatorial explosion. Under those circumstances, an alternative approach such as Markov Chain may be advised. However, where the number of threat motion hypotheses is kept to fewer than a dozen, Monte Carlo simulation is a suitable modeling framework, even on a slow desktop computer. Lastly, Monte Carlo simulation is an attractive framework for ASW analysis because it is easy to work with and because it can handle complex submarine motion objectives with suitable fidelity.

3.2 Bayesian Inferencing

The Bayes Theorem, formula 3 [7], is well suited to the application of modeling uncertainty as it applies the ASW effectiveness of a distributed multistatic field. There are three interrelated attributes that make Bayes Theorem very desirable for this application. First, it is constructed to deal with negative information. Since normative ASW involves searching while there is no target within acoustic range of the sensors, having a means to "learn" from that condition can be quite valuable. Second, the formulation as a conditional probability is well suited for evaluating concurrent and competing hypotheses. Third, Bayes Theorem normalizes the implication of the negative search information (i.e., no detection). This last point means that negative search does not diminish the probability that the target is somewhere within the analytical purview of the algorithm. When Bayes Theorem is implemented properly, this attribute has great utility, but when implemented improperly, it becomes a liability.

$$P[A_j \mid B] = \frac{P[B \mid A_j] * P[A_j]}{\sum_{i=1}^{n} P[B \mid A_i] * P[A_i]} \qquad (3)$$

Where:

$P[A_j|B]$ - is the probability that the target is in A_j given search event B failed to detect it, the *posterior probability*.

$P[B|A_j]$ - is the probability of target remaining undetected in area A_j given search event B, the *likelihood function*.

$P[A_j]$ - is the probability of target being in area A_j.

Denominator is the *marginal probability* or the *normalizing constant* represented as the sum of all mutually exclusive hypotheses.

3.2.1 Low Probability "Looks"

So far, this paper has argued for the perspective that exact sensor performance is not anticipatable. On any given source-target-receiver look-event, it is not reasonable to ask whether or not the target would be detected; it is only reasonable to ask what is the probability that the target would be detected. It is possible to envision systems and field designs where performance would intentionally be based on multiple "looks" each having a probability of detection of less than 0.5. It would not be suitable for a performance prediction engine to round up all "looks" greater than 0.5 probability and dismiss all "looks" less than 0.5 probability. To address this issue, a Bayesian framework may be adopted.

Alternatively, a system could simply Monte Carlo the probability of detection for each "look", such that 20% of the time a 0.2 probability of detection "look" will result in a modeled detection and the Monte Carlo track eliminated. This approach has the effect of forcing certain outcomes onto uncertain data. One limitation of this approach is that the analyst cannot readily discern if the tracks that escaped detection were "lucky" tracks or tracks that the system never even looked at. Another limitation of this approach is discussed in the Positive Data Fusion section of this paper; when tracks are eliminated, they are no longer available to future information fusion. It is best to keep all Monte Carlo target tracks and still have a way to account for the effects of search histories that may be comprised of multiple low probability of detection "looks".

3.2.2 Learning from Negative Search

Much of ASW takes place without a threat submarine actually operating in the acoustic range of searching sensors. While the absence of a submarine may be undesirable in an experimental setting, it may be very desirable in time of war. The challenge is to discern whether or not a threat submarine is or may be present. Conducting ASW search with the purpose of discerning that a submarine threat is not present is known as Area Clearance and it must have associated with it a particular level of confidence. Bayesian inferencing is an excellent way to infer meaning to search that occurs without detection on the threat submarine [8]; this is often referred to as negative search information. In a Bayesian search engine, it is presumed that search effort results in no detections until some overt act is undertaken to infuse a detection or contact report. Negative search information is accreted against each Monte Carlo track.

There are differences in how various Bayesian systems work; the below description applies to MUSICAL™. The history of search events is used to calculate the probability that each Monte Carlo track would have remained undetected. This is used to adjust the credibility of each of the Monte Carlo tracks, which is referred to as its "weight". As described so far, this would result only in the diminishment of target existing, if the weight of each Monte Carlo could only be diminished as the consequence of negative search. But this is not what happens in a Bayesian framework. The Bayesian framework is a closed system. If a submarine is said to exist at the beginning of the analytical problem, it must also exist at the end of the analytical problem. The weights of the tracks are normalized at the end of each look event by the denominator of Formula (2). Through normalization of the Monte Carlo weights, the probability of the target existing remains constant, while at the same time the relative weight of the submarine is being shifted from the tracks that are under the influence of the multistatic field to tracks that have not been searched.

3.2.3. Positive Data Fusion

A Bayesian fusion method for calculating the threat density probability map (TDPM) in MUSICAL™ has the advantage of retaining Monte Carlo tracks even when they may have very low probabilities. This is very useful when a detection or contact report arrives (i.e., positive search data). The objective is to fuse the detection/contact report with the *a priori* TDPM. The contact report may have its own distribution function (e.g., uniform or bivariate normal) and a credibility. A type of normalized cross-product between the *a priori* and the contact report produces the new TDPM. In a non-Bayesian framework there is the risk that the *a priori* density would have removed all the Monte Carlo tracks from the region in which contact was gained. A Bayesian framework not only preserves the tracks but also preserves the differences between the low probability tracks such that the normalized cross-product no longer is uniform or bivariate normal in the region of the contact report. In other words, the data in the contact report is informed by the previous search history. This is especially valuable when the contact report covers a large area of uncertainty that falls over an area where the *a priori* TDPM is non-uniform.

4. Exemplar Analysis

Two exemplars are offered in this section to help demonstrate what a combined Monte Carlo and Bayesian framework might look like, when used to make performance predictions for a distributed multistatic sensing field. The two exemplars are Area Clearance and an Area Denial.

4.1 Area Clearance

Area Clearance is the ability to claim with some measure of confidence that, based on some previous or ongoing

search effort, a threat submarine is not likely to be operating in a specified region. In the absence of a negative search inference engine, such as Bayes Theorem, analysts are left with two alternatives, neither of which is desirable. One approach is for the analyst to make an educated guess. There are tactical problems for which this is possible. However, distributed multistatic sensor fields in a range dependent environment that exhibit spatial and temporal discontinuities in performance should not be treated as an intuitive problem. An alternative approach that has been used is to create an impermeable analytical boundary around the sensor field, and by one means or another calculate the cumulative probability of detection against any target operating within. This is an undesirable approach to the problem because "edge effects" often drive the real effectiveness of the distributed field. The impermeable boundary means that the entire search effort (time) will be applied to all target tracks in the analysis space. This ignores the effect of a submarine entering the field after the search has begun or leaving the field before it is completed. Ignoring the edge effect may seriously compromise the analysis.

Figure 3 shows the initial conditions of the Area Clearance exemplar. A total of 6,000 Monte Carlo track representations of the enemy submarine have been randomly placed within a 100 NM by 100 NM area that contains some land features. Five stationary sources have been placed in the battlespace and four mobile receivers are available to receiver bistatic echoes and share information with one another. Analysis, not shown in this report, was conducted to evaluate if the receivers should be co-located with the four corner sources (i.e., monostaticly placed) or whether bistatic placement would be better. Analysis showed that field performance was greater when the mobile receivers where bistaticly positioned. Optimization analysis (i.e., optimal of all tactics evaluated) showed that the some candidate tactics placed the mobile receivers too close to the sources while other tactics placed them too far from the sources. Figure 3 shows the results of this optimization analysis with the mobile receivers circumnavigating the four sources at some intermediate range from them. These results are very much dependent on the range dependent sensor performance predictions pertaining to this environment, target, and multistatic system.
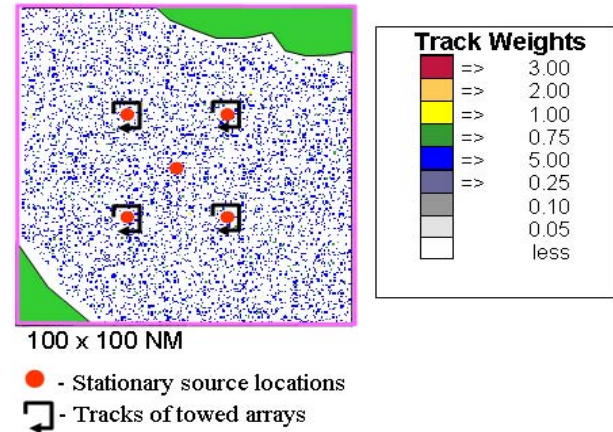

100 x 100 NM

● - Stationary source locations

⌐┘ - Tracks of towed arrays

Figure 3. Threat Density Probability Map Hour 0

Figure 4 shows the impact that 16 hours of search without contact has on the initial assumption of a uniformly distributed target. The central part of the figure seems to have no Monte Carlo tracks within it. For this display, Monte Carlo tracks that had a greater than 0.95 probability of having been detected were colored white, causing them to disappear. This convention was adopted because those tracks are deemed improbable and should not distract the operator.

The effects of the range dependent environment on the performance can be seen in the TDPM. The clearance around the southeast source is less than the clearance around the southwest source. Additionally, there is path of cleared water extending out of the field to the south. This is all driven by range dependent sensor performance.

A 40 by 40 NM gray box has been plotted on the 16 hour TDPM indicating the most effectively cleared region. Given the target motion assumptions and the range dependent multistatic performance predictions, there is less than a 1% chance that target would be in the gray region and undetected. In other words, it represents the region having the least residual ASW risk. It should be observed that the most effectively cleared region is not in the center of the formation. A tactician relying on a cumulative probability of detection metric does not gain this insight. An acoustician looking at range dependent performance may understand that performance is skewed to the south and west, but still could not assess the benefit of preferentially placing a high value unit there. This exemplar shows that a combination of Monte Carlo simulation and Bayesian inferencing methods can achieve this end.
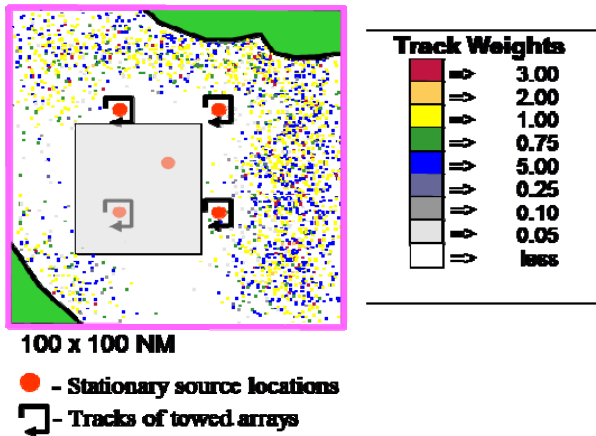
Track Weights

(red)	⇒	3.00
(orange)	⇒	2.00
(yellow)	⇒	1.00
(green)	⇒	0.75
(blue)	⇒	5.00
(gray-blue)	⇒	0.25
(gray)	⇒	0.10
(light gray)	⇒	0.05
(white)	⇒	less

100 x 100 NM

● - Stationary source locations

⤶ - Tracks of towed arrays

Figure 4. TDPM After 16 Hours of Search with Gray Box Indicating Best Cleared Area 40X40 NM Box

4.2 Area Denial

Area Denial is the ability to detect, classify, and respond to enemy submarines in a timely fashion as they attempt to enter into some battlespace that is being controlled. The nature of the response and what is meant by "timely" is a function of mission, threat capability, and friendly force capability. Area Denial is a logical follow-on to Area Clearance. Area Denial may employ the same resources and tactics that were used in Area Clearance or a different tactic might be employed. For example, a barrier search might be established along each of the four sides of the cleared battlespace.

Figure 5 shows an analytical scenario where the Area Clearance tactic is maintained to provide Area Denial capability. There are three submarine approach directions that are considered credible. Monte Carlo simulation is well suited to the creation of three concurrent threat motion hypotheses. In this case, each hypothesis is considered equally likely, and so an equal number of Monte Carlo tracks are allocated to each of three. The analytical objective is to identify tactics that provide the greatest effectiveness against the three equally likely hypotheses. As a matter of efficiency, it is best to evaluate these three hypotheses concurrently. When complex, responsive behaviors are modeled, it may become necessary to model the hypotheses concurrently.
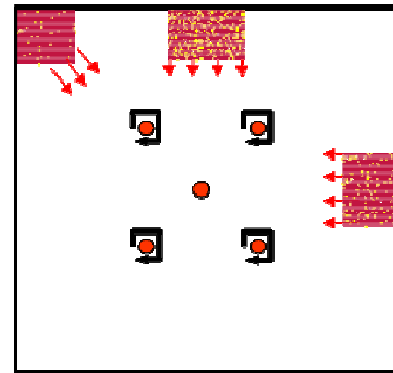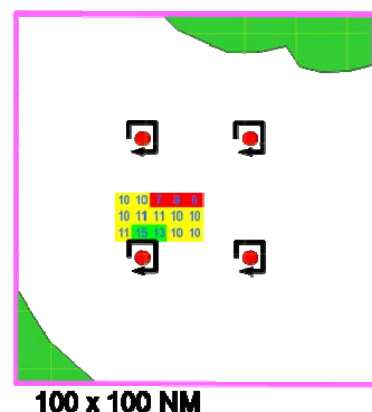

Figure 5. Multiple Hypotheses for Area Denial

The objective of this exemplar is to detect, classify and localize the threat submarine prior to it reaching a circle of 20 NM radius. The placement of the circle will be governed by the above metric. The search plan will remain unaltered, regardless of the placement of the circle within the battlespace. The threat will be modeled as having knowledge of the circle location and the Monte Carlo tracks will drive towards the circle regardless of its placement.

Figure 6 shows the relative results of 15 candidate locations for the 20 NM radius circle. When centered on the middle of formation, the field is effective against the northwest hypothesis (worth 5 points) and is minimally effective against the north hypothesis (worth 1 point) and is completely ineffective against the east hypothesis. The score of 6 points is colored red to indicate that centering the circle in the center of the formation is poor choice. There are two locations colored green, indicating that they are effective against all three hypotheses. To the west of the green location with 15 points (the maximum score considered), the score drops off again. This is because as the circle migrates too far to the west, the northwest hypothesis is able to approach the circle while circumnavigating the acoustic effectiveness of the multistatic field.

100 x 100 NM

● - Stationary source locations

⤶ - Tracks of towed arrays

Figure 6. Optimal Placement of Circle Center

Search against each of the three hypotheses will show that each is unlikely. If these are the only hypotheses being evaluated, the combined weight of the three hypotheses will eventually be shifted to the east hypothesis as it is the most difficult to disprove. If a fourth concurrent hypothesis were created in which the threat was said to be anywhere in the region and in a random patrol (Figure 3), most of the weight of the first three hypotheses would eventually be shifted to fourth hypothesis. Eventually, the TDPM would look just like Figure 4.

Conclusions

Monte Carlo simulation techniques combined with Bayesian integration methods provide a robust framework for analyzing the performance of distributed multistatic systems. This combination can be used to account for model inputs that are unknown and/or fluctuating and it can produce unique information products that are meaningful to the ASW decision-maker.

Monte Carlo simulation can account for tactical uncertainty with respect to target motion and motion objectives in a three dimensional battlespace. Monte Carlo techniques also allow for the modeling of concurrent, competing hypotheses.

Bayesian integration methods supports the reevaluation of competing hypotheses such that as search is conducted, some hypotheses are evaluated as less likely and other hypotheses are evaluated as more likely.

Bayesian integration methods also support the creation of threat density probability maps, which are quantitative and intuitive representations of the combined effect of assumed target behavior and search history. The threat density probability maps allow quantifying the degree to which an area has been cleared and what area within the searched region is most effectively cleared.

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